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Deeply exclusive processes and generalized parton distributions

Marc Vanderhaeghen[1,2]

[1] Thomas Jefferson National Accelerator Facility, Newport News, VA 23606, USA
[2] Physics Department, College of William and Mary, Williamsburg, VA 23187, USA

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Abstract. We discuss how generalized parton distributions (GPDs) enter into hard exclusive processes, and focuss on the links between GPDs and elastic nucleon form factors. These links, in the form of sum rules, represent powerful constraints on parametrizations of GPDs. A Regge parametrization for the GPDs at small momentum transfer $-t$ is extended to the large-t region and it is found to catch the basic features of proton and neutron electromagnetic form factor data. This parametrization allows to estimate the quark contribution to the nucleon spin. It is furthermore discussed how these GPDs at large-t enter into two-photon exchange processes and resolve the discrepancy between Rosenbluth and polarization experiments of elastic electron nucleon scattering.

PACS. 12.38.Bx Perturbative calculations – 13.60.Fz Elastic and Compton scattering

1 Introduction

Generalized parton distributions (GPDs) are universal non-perturbative objects entering the description of hard exclusive electroproduction processes (see [1,2,3,4] for reviews and references). These GPDs depend upon the different longitudinal momentum fractions of the initial (final) quark and upon the overall momentum transfer t to the nucleon.

As the momentum fractions of initial and final quarks are different, one accesses quark momentum correlations in the nucleon. Furthermore, if one of the quark momentum fractions is negative, it represents an antiquark and consequently one may investigate $q\bar{q}$ configurations in the nucleon. Therefore, these functions contain a wealth of new nucleon structure information, generalizing the information obtained in inclusive deep inelastic scattering.

In particular, the t-dependence of the GPDs has attracted a considerable interest recently. It has been shown that by a Fourier transform of the t-dependence of GPDs, it is conceivable to access the distributions of parton in the transverse plane, see [5], and to provide a 3-dimensional picture of the nucleon [6]. The t-dependence of moments of GPDs have also become amenable to lattice QCD calculations [7] recently.

2 Regge parametrization of GPDs and link to the elastic form factors

The t-dependence of the GPDs is directly related to nucleon elastic form factors (FFs) through sum rules. In particular, the nucleon Dirac and Pauli form factors $F_1(t)$ and $F_2(t)$ can be calculated from the GPDs H and E through the following sum rules for each quark flavor ($q = u, d$)

$$F_1^q(t) \;=\; \int_{-1}^{+1} dx\; H^q(x, \xi, t)\,, \tag{1}$$

$$F_2^q(t) \;=\; \int_{-1}^{+1} dx\; E^q(x, \xi, t)\;. \tag{2}$$

One can choose $\xi = 0$ in the previous equations, and model $H(x, 0, t)$ and $E(x, 0, t)$. In modeling the t-dependence of GPDs, the Regge picture suggests a behavior for the GPDs at small x as [3] :

$$H^q(x, \xi = 0, t) = q_v(x) x^{-(\alpha(t) - \alpha(0))}, \tag{3}$$

with q_v the forward valence quark distributions. Assuming a linear Regge trajectory with the slope $\alpha^{'}$, one gets

$$H^q_{Regge}(x, 0, t) = q_v(x) x^{-\alpha_1^{'} t}, \tag{4}$$

The notation $\alpha_1^{'}$ emphasizes that this is the slope of the leading Regge trajectory for the F_1 form factor.

The Dirac mean squared radii of proton and neutron in this model are given by

$$r^2_{1,p} \;=\; -6\, \alpha_1^{'} \int_0^1 dx \left\{ e_u\; u_v(x) \,+\, e_d\; d_v(x) \right\} \ln x\;, \tag{5}$$

$$r^2_{1,n} \;=\; -6\, \alpha_1^{'} \int_0^1 dx \left\{ e_u\; d_v(x) \,+\, e_d\; u_v(x) \right\} \ln x\;, \tag{6}$$

with $e_u = +2/3$ and $e_d = -1/3$, which yield for the electric mean squared radii of proton and neutron :

$$r^2_{E,p} \;=\; r^2_{1,p} \,+\, \frac{3}{2}\,\frac{\kappa_p}{m_N^2}\;, \qquad r^2_{E,n} \;=\; r^2_{1,n} \,+\, \frac{3}{2}\,\frac{\kappa_n}{m_N^2}\;, \tag{7}$$

where κ_p (κ_n) are the proton (neutron) anomalous magnetic moments. For the proton, the rms radius $r^2_{E,p}$ requires the value $\alpha_1^{'} = 1.0 - 1.1$ GeV^{-2}. Such a value is close to the expectation from Regge slopes for meson trajectories, therefore supporting the ansatz of (4).

To calculate F_2, one needs a parametrization for the nonforward parton densities $E^q(x, 0, t)$. The Regge picture suggests a similar type structure

$$E^q_{Regge}(x, t) = e^q(x)\, x^{-\alpha_2^{'} t}, \qquad (8)$$

as for $H^q(x, 0, t)$, with possibly a slightly different slope $\alpha_2^{'}$. The next step is to model the forward magnetic densities $e^q(x)$. The simplest idea, is to take them proportional to the $q_v(x)$ densities as :

$$e^u(x) = \frac{\kappa_u}{2} u_v(x) \qquad \text{and} \qquad e^d(x) = \kappa_d d_v(x)\ , \qquad (9)$$

satisfying the normalization conditions

$$\kappa_q \equiv \int_0^1 dx\; e^q(x), \qquad (10)$$

which, in their turn, guarantee that $F_2^p(0) = \kappa_p$, and $F_2^n(0) = \kappa_n$, using $\kappa_u = 2\kappa_p + \kappa_n$ and $\kappa_d = \kappa_p + 2\kappa_n$.

The above model fits the proton elastic form factor data for small $-t < 0.5\,\mathrm{GeV}^2$ but falls considerably short of the data for $-t > 1\,\mathrm{GeV}^2$.

To improve the agreement with the precise data for nucleon form factors at large $-t$ which have been obtained in recent years, both a Gaussian-type model, as first discussed in [8], and the Regge-type model discussed above have to be modified. The shortcomings of the gaussian and Regge models are that they do not satisfy the Drell-Yan-West (DYW) relation [9,10] between $x \to 1$ behavior of the structure functions and the t-dependence of elastic form factors. According to DYW, if the parton density behaves like $(1-x)^\nu$, then the relevant form factor should decrease as $1/t^{(\nu+1)/2}$ for large t. The simplest idea to satisfy DYW and to preserve the Regge structure at small x and t is through the modified Regge ansatz [11]

$$H^q_{mod.Regge}(x, 0, t) = q_v(x) x^{-\alpha_1^{'}(1-x)t}\ . \qquad (11)$$

In the following estimates, the unpolarized parton distributions from the MRST2002 global NNLO fit [12] are used. This leads to a $x \to 1$ behavior as $q_v(x) \sim (1-x)^{\nu_q}$, with $\nu_u = 3.50$ and $\nu_d = 4.03$ at a scale $\mu^2 = 1$ GeV^2.

In case of the Pauli form factor F_2, the same modification of the Regge ansatz yields

$$E^q_{mod.Regge}(x, 0, t) = e^q(x) x^{-\alpha_2^{'}(1-x)t}\ . \qquad (12)$$

Experimentally, the proton helicity flip form factor $F_2(t)$ has a faster power fall-off at large t than $F_1(t)$. Within the modified model, this means that the $x \sim 1$ behavior of the functions $E(x, 0, t)$ and $H(x, 0, t)$ should be different. To produce a faster decrease with t, the $x \to 1$ limit of the density $E^q(x, 0, t)$ should have extra powers of $1 - x$ compared to that of $H^q(x, 0, t)$. Without introducing too many free parameters, this can be achieved by multiplying the valence quark distributions in the ansatz for $e^q(x)$ by an additional factor $(1-x)^{\eta_q}$, i.e.,

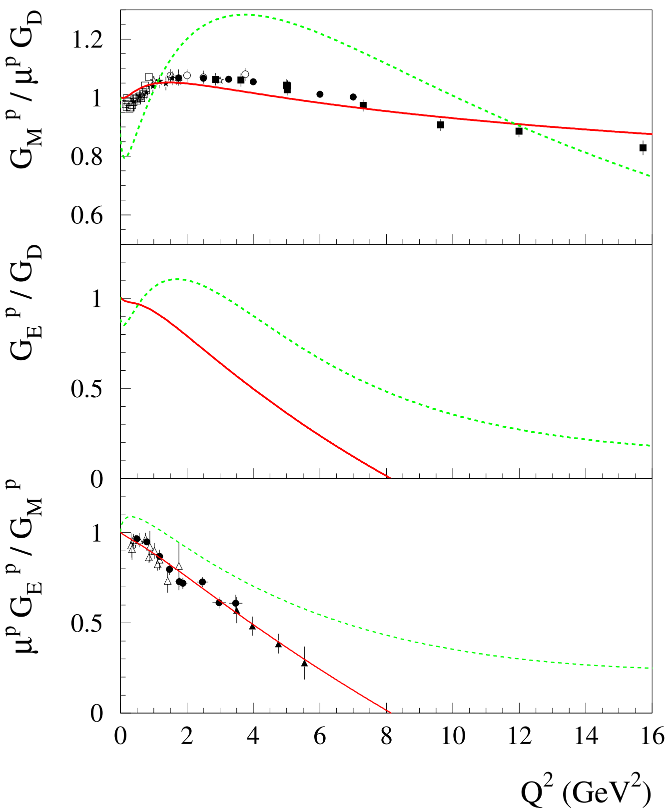

Fig. 1. Proton magnetic (*upper panel*) and electric (*middle panel*) form factors compared to the dipole form $G_D(t) = 1/(1-t/0.71)^2$, as well as the ratio of both form factors (*lower panel*). The *solid curves* correspond to the modified Regge parametrization with $\alpha_1^{'} = 1.098$ GeV^{-2}, $\alpha_2^{'} = 1.158$ GeV^{-2}, $\eta_u = 1.52$ and $\eta_d = 0.31$. For comparison also the result for a Gaussian model as in [8] is shown (*dashed curves*). The references to the data can be found in [11]

$$e^u(x) \;=\; \frac{\kappa_u}{N_u}(1-x)^{\eta_u} u_v(x)\,, \qquad (13)$$

$$e^d(x) \;=\; \frac{\kappa_d}{N_d}(1-x)^{\eta_d} d_v(x)\ , \qquad (14)$$

where the normalization factors N_u and N_d guarantee the conditions (10). The powers η_u and η_d are to be determined from a fit to the proton form factor data. Note that the value $\eta_q = 2$ corresponds to $1/t$ asymptotic behavior of the ratio $F_2^q(t)/F_1^q(t)$ at large t.

In Figs. 1 and 2, the proton and neutron Sachs electric and magnetic form factors are shown, which are obtained from F_1, F_2 as $G_E = F_1 - \tau F_2$, $G_M = F_1 + F_2$ with $\tau \equiv -t/4M_N^2$. One observes that the modified Regge model gives a very good description of all available form factor data for both proton and neutron.

Since the GPD E enters the sum rule for the total angular momentum J^q carried by a quark of flavor q in

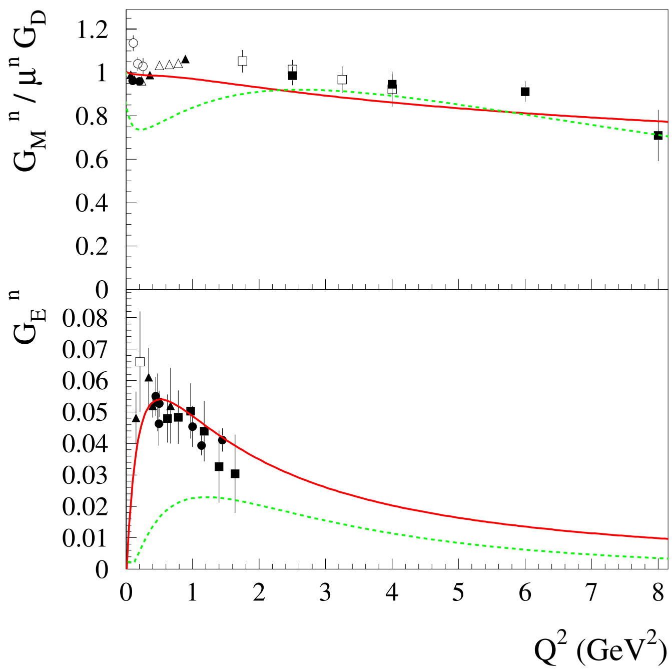

Fig. 2. Neutron magnetic form factor compared to the dipole form (*upper panel*), and neutron electric form factor (*lower panel*), with curve conventions as in Fig. 1. The references to the data can be found in [11]

the proton as [13] :

$$2J^q = \int_{-1}^{1} dx\, x\, \{H^q(x,0,0) + E^q(x,0,0)\}\,, \qquad (15)$$

our parametrization in which the $x \to 1$ limit of E is determined from the F_2^p/F_1^p form factor ratio, allows to evaluate the above sum rule. The first term in the sum rule of (15) is already known from the forward parton distributions and is equal to the total fraction of the proton momentum carried by a quark of flavor q ($q = u, d, s$) :

$$M_2^q \equiv \int_{-1}^{1} dx\, x H^q(x,0,0) = \int_{0}^{1} dx\, x\, [q_v(x) + 2\,\bar{q}(x)]\, (16)$$

with $\bar{q}(x)$ the anti-quark distribution. The 'non-trivial' contribution to the sum rule arises from the second moment of the GPD E. In Table 1, the values of the quark momentum sum rule M_2^q at the scale $\mu^2 = 1$ GeV^2 are shown, as well as the modified Regge estimate for J^u, J^d, and J^s. This estimate leads to a large fraction (63 %) of the total angular momentum of the proton carried by the u-quarks and a relatively small contribution carried by the d-quarks. As the d-quark intrinsic spin contribution is known to be relatively large and negative ($\Delta d_v \simeq -0.25$), the small total angular momentum contribution J^d of the d-quarks which follows from our parametrization implies an interesting cancellation between the intrinsic spin contribution and the orbital contribution L^d (with $2J^q = \Delta q + 2L^q$), which should therefore be of size $2L^d \simeq 0.2$. For the u-quark on the other hand, the estimate for $2J^u$ is quite close to the intrinsic spin contribution $\Delta u_v \simeq 0.6$. Such a picture is also supported by

Table 1. Estimate of $2\,J^q$ (second column) for the different quark flavors at the scale $\mu^2 = 1$ GeV^2 according to (15), using the modified Regge parametrization for the GPD E. For the forward parton distributions, the MRST2002 NNLO parametrization [12] for M_2^q (first column) is used. For comparison, the third column shows the quenched lattice QCD results of [14], extrapolated to the physical pion mass

	M_2^q	$2\,J^q$ (mod. Regge)	$2\,J^q$ (lattice [14])
u	0.40	0.63	0.734 ± 0.135
d	0.22	-0.06	-0.085 ± 0.088
s	0.03	0.03	
$u+d+s$	0.65	0.60	0.65 ± 0.16

a recent quenched lattice QCD calculation [14]. One indeed sees from Table 1 that the quenched lattice QCD calculation yields quite similar values for $2J^u$ and $2J^d$ as the modified Regge parametrization. It remains to be seen however how large is the sea quark contribution to the GPD E which can enter the spin sum rule of (15). This sea quark contribution is absent in the quenched lattice QCD calculations of [14]. It is also not constrained by the form factor sum rules, which only constrain the valence quark distributions. Ongoing measurements of hard exclusive processes provide a means to address this contribution.

3 Two-photon exchange in elastic electron–nucleon scattering at large $-t$

The nucleon electromagnetic form factors as discussed above have been usually extracted using elastic electron-nucleon scattering in the one-photon exchange approximation. One surprising experimental result from polarization experiments of elastic electron-nucleon scattering has been the ratio of electric to magnetic proton form factors, which is clearly at variance with unpolarized measurements using the Rosenbluth separation technique.

The understanding of this puzzle has generated a lot of activity very recently. It has been suggested in various calculations and analyses that contributions from two-photon exchange amplitudes at the level of a few percent can help resolve this puzzle. The general structure of two- (and multi)-photon exchange contributions to the elastic electron proton scattering observables has recently been studied [15]. It was found in that work that the 2γ exchange contribution to the unpolarized cross section can be kinematically enhanced at larger Q^2 compared with the $(G_{Ep})^2$ term, while the 2γ exchange contribution to the polarization measurements need not affect the results in a significant way. Explicit model calculation of the 2γ exchange effects were performed recently in [16,17].

In [17], the elastic ep scattering has been evaluated at large momentum transfer through the scattering off partons in a nucleon. In particular, the two-photon exchange amplitude has been related to the GPDs at large $-t$, as discussed above. This partonic calculation involves

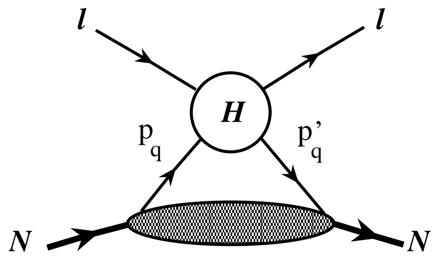

Fig. 3. Handbag diagram for the elastic *ep* scattering at large momentum transfers. H is the hard scattering process, whereas *lower blob* represents the GPDs of the nucleon

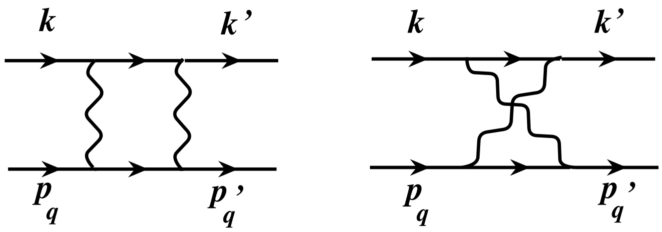

Fig. 4. Direct and crossed box diagrams for H in Fig. 3

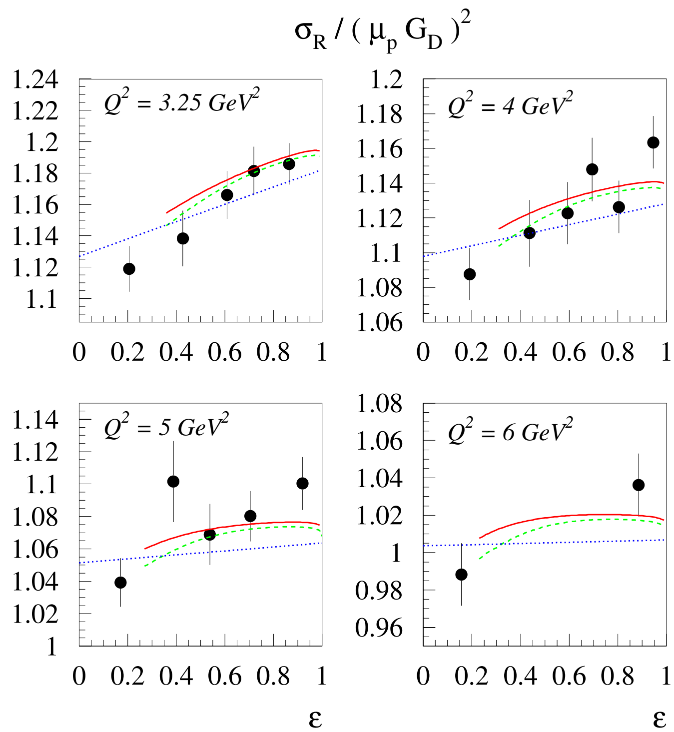

Fig. 5. Rosenbluth plots for elastic *ep* scattering : σ_R divided by $(\mu_p G_D)^2$, with $G_D = (1 + Q^2/0.71)^{-2}$. *Dotted curves:* Born approximation using G_{Ep}/G_{Mp} from polarization data. The results when adding the GPD calculation for the 2γ exchange correction, are shown for the gaussian (*dashed curves*) and modified Regge (*solid curves*) GPD parametrizations

the evaluation of the handbag diagram of Fig. 3. The hard scattering kernel entering in this diagram is shown in Fig. 4, corresponding with the box diagram at the quark level. As a result of the evaluation of the handbag diagram of Fig. 3, the two-photon exchange amplitude for elastic electron-nucleon scattering at large -t can be expressed in terms of 3 integrals containing large -t GPDs (see [17] for details).

In Fig. 5, we display the effect of 2γ exchange on the cross sections. Fig. 5 illustrates that the values of G_{Ep} as extracted from the polarization data are inconsistent with the slopes one extracts from a linear fit to the Rosenbluth data in the Q^2 range where data from both methods exist. By adding the 2γ correction, one firstly observes that the Rosenbluth plot becomes slightly non-linear, in particular at the largest ε values. Furthermore, one sees that over most of the ε range, the slope is indeed steeper in agreement with the Rosenbluth data. One sees that including the 2γ exchange allows to reconcile both polarization transfer and Rosenbluth data.

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